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Overview and Purpose:

The rationale for offering a "Core-Aligned" College Algebra course stems from the need to ease the transition for incoming students from our regional/local high schools to collegiate mathematics. We are not suggesting a complete redesign of our current course. Instead, we provide optional, supplemental activities that can be easily integrated into our current course structure, but that provide students more in-depth opportunities to explore the concepts. In the age of Common Core State Standards for Mathematics (CCSSM), students expect mathematics courses to be focused on problem-solving, modeling of authentic contexts, and conceptual understanding, yet many of our College Algebra topics are taught without these considerations. These supplements offer instructors of College Algebra accessible ways to bring the "spirit" of the Common Core to our traditional course, while still maintaining the elements with which we are most accustomed.

Mathematical Practices:

The CCSSM offer a focus on eight mathematical practices across all grades K–12. These practices "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (CCSSM, p. 6). Two of those standards state students should "reason abstractly and quantitatively" (p. 6) and "make sense of problems and persevere in solving them" (p. 6). To bring the spirit of the Common Core to our College Algebra course, we focus on these two mathematical practices.

The Importance of Reasoning and Sense-Making in Mathematics:

As a result of Tennessee's adoption of the CCSSM, both students and mathematics instructors are raising their expectations for the teaching and learning of mathematics. As such, reasoning and problem solving must become more central in our approaches to the teaching of key mathematics content at the college level. According to the National Research Council's 2001 report, *Adding It Up*, mathematical proficiency "captures what we believe is necessary for anyone to learn mathematics successfully" (p. 116) and two of those five strands of mathematical proficiency are *strategic competence* and *adaptive reasoning*. These proficiencies are crucial because "strategic competence comes into play at every step in developing procedural fluency in computation" (p. 124) and "adaptive reasoning is the glue that holds everything together, the lodestar that guides learning" (p. 129). "Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts" (NCTM, 2000, p. 56). According to Bass and Ball (2003), "What, after all, would mathematical "understanding" mean if it were not founded on mathematical reasoning?" (p. 28). We, as post-secondary mathematics instructors, must embrace and facilitate reasoning, sense-making, and problem solving in our college algebra courses. These "core-aligned" supplements to College Algebra offer that opportunity.

Appropriate Use of Technology:

Technology is encouraged in the teaching of College Algebra, given that it is used appropriately–– to facilitate student learning, not to replace student thinking and/or computational fluency. Graphing calculators and online applets can offer students opportunities to explore functions in ways that are more engaging and more conceptual than non-technology enhanced options. It is important, however, to maintain an appropriate balance between the use of technology and the development of computational skills.

References

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, G. W. Martin, & D. Schifter (Eds.), *A research companion to the principles and standards for school mathematics* (pp. 27–44). Reston, VA: The National Council of Teachers of Mathematics.

Carpenter, J., Gorg, S., & Martin, G. W. (Eds.). (2000). *Principles and standards for school mathematics*. Reston, VA: The National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). Common core state standards: Mathematical practice. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers.

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CORE TO COLLEGE INITIATIVE
COURSE SYLLABUS
{CORE-ALIGNED} COLLEGE ALGEBRA

Class Hours: 3.0 **Credit Hours: 3.0**

Laboratory Hours: 0.0 **Revised: Fall 2013**

Catalog Description:
Model and apply linear, polynomial, exponential, power, piece-wise, and logarithmic functions to make sense of, and to solve problems.

Entry Level Standards:
Students must be able to read at the college level. *<Insert other institutional standards here>*

Prerequisites:

Successful completion of four high school mathematics courses that include Algebra 1, Geometry, and Algebra II; and PARCC score of *x* (college and career ready), or Compass score of *y*.

Textbook(s) and Other Course Materials:

<Insert textbook/course materials description here>

Textbook:

<Insert textbook here if there is one that meets your course description>

Technology Requirement:

A graphing calculator such as TI 83/84, TI-Nspire, or TI-Nspire CX. Calculators with a computer algebra system (CAS) are not permitted.

Course Description:
{Core-Aligned} College Algebra provides students an in-depth study of modeling and applying functions. Course applications originate from home, work, recreation, consumer issues, public policy, and scientific investigations. Appropriate technologies and software are used regularly for instruction and assessment.

1. Use mathematics to solve problems and determine if the solutions are reasonable.
2. Use mathematics to model real world behaviors and apply mathematical concepts to the solution of real-life problems.
3. Make meaningful connections between mathematics and other disciplines.
4. Use technology for mathematical reasoning and problem solving.
5. Apply mathematical and/or basic statistical reasoning to analyze data and graphs.

While the overall goal of this course is to use data to motivate deeper understanding of functions, non-data modeling should not be excluded.

Essential Questions:

1. How, and why, do the terms and symbols that are unique to algebra enable us to read, write, speak about, and understand mathematics?
2. How, and why, do we use technology to analyze data and develop mathematical models?
3. How, and why, do we use a graphing calculator and other technologies to understand the nature and behavior of functions?
4. How, and why, do we model and solve problems using multiple representations (equations, graphs, tables, words)?

I. Goals and Objectives

Goal: Analyze data and apply concepts to solve problems.

Objective 1: Model and solve problems using tables, graphs, and algebraic properties with power functions, polynomial and rational functions, exponential and logarithmic (common, natural) functions, and piecewise-defined functions. Interpret constants, coefficients, and bases in the context of the problem.

Students will demonstrate the ability to:

Outcome 1.1: Describe graphically, algebraically and verbally phenomena as functions; identify independent and dependent quantities, domain, and range, and input/output.

Outcome 1.2: Define and use advanced functions to model problems, make inferences, and justify results.

Outcome 1.3: Extend knowledge of mathematics through relevant mathematical modeling with applications, problem solving, critical thinking skills, and the strategic use of appropriate technologies.

Supplements/Activities:

End-Behavior of Rational Functions

The Importance of Horizontal Asymptotes

Presenting Data (What's the "Big Whoop?")

Objective 2: Use systems of two or more equations and/or inequalities to solve problems using tables, graphs, and algebraic properties. Interpret intersections/regions in the context of the problem.

Students will demonstrate the ability to:

Outcome 2.1: Describe graphically, algebraically and verbally phenomena modeled by systems of two or more equations or inequalities.

Outcome 2.2: Identify intersections/regions in the context of the problem.

Outcome: 2.3: Extend knowledge of mathematics through relevant mathematical modeling with applications, problem solving, critical thinking skills, and the strategic use of appropriate technologies.

Supplements/Activities:

Applications of Systems of Linear Equations with Infinitely Many Solutions (Traffic Flow)

Objective 3: Create and use calculator-generated models of linear, polynomial, exponential, rational, power, and logarithmic functions of bivariate data to solve problems.

Students will demonstrate the ability to:

Outcome 3.1: Interpret the constants, coefficients, and bases in the context of the data.

Outcome 3.2: Use the most appropriate model to draw conclusions and make predictions.

Outcome 3.3: Visually inspect models for goodness-of-fit.

Outcome 3.4: Apply arithmetic, algebra, higher-order thinking and statistical methods to modeling and solving real-world problems.

Supplements/Activities:

Culminating Project: Creating Models for Data

II. Week/Unit/Topic Basis:

Applications and modeling of data should be interwoven throughout all of the topics below.

We leave pacing decisions to the course instructor, but suggest the following sequence of topics:

- Introduction to Modeling with a focus on Linear Functions
- Power and Piecewise Functions
- Polynomial and Rational Functions
- Exponential and Logarithmic Functions
- Systems of Equations/Inequalities

The most significant portion of the course should be dedicated to functions. If your College Algebra course includes systems of equations/inequalities, then you would incorporate Objective 2 and the *Traffic Flow* Supplement.

III. Evaluation:

A. Pre-Assessment

We encourage the use of pre-assessments so that material might be skipped and/or emphasized based on your students' needs.

B. Formative/Summative Assessment

The use of both formative and summative assessments is important in the context of this course (see supplemental course materials for ideas).

C. Grading

A suggested weighting for the course grade is listed, but instructors should follow standards set by their departments.

Supplements	20%
Homework	15%
Three Exams (15% each)	45%
Comprehensive Final Exam	20%

D. Other Evaluation Methods:

As assigned by instructor

E. Grading Scale:

As assigned by instructor

Supplementary Task: Examining the End Behavior of Rational Functions

Students often do not understand the connection between the end behavior of polynomials and the end behavior of rational functions. Often, discussions of rational functions in basic college algebra courses center on graphing the function, paying particular attention to finding the zeroes and asymptotes of the function. In many cases, students are given a set of "rules" for finding the horizontal asymptote or slant asymptote (if either one exists) with no context of why these "rules" are valid. This task is designed to support students develop a deeper understanding of the graph of a rational function be examining the end behavior and should be used near the beginning of the study of rational functions.

These examples support the objective:

> Use functions to model and solve problems using tables, graphs, and algebraic properties. Interpret constants, coefficients, and bases in the context of the problem.

in the Core to College Master Syllabus for Core-Aligned College Algebra.

Example: Examining the End Behavior of Rational Functions

For each of the following rational functions:

i. Rewrite the rational function as the sum of a polynomial and a rational function whose numerator has a smaller degree than its denominator. (Remember, a polynomial can be a constant.)
ii. Graph the original rational function using your graphing calculator.
iii. Graph the polynomial portion of your rewritten form of the original rational function.
iv. Discuss any relationships you see between the end behavior of the polynomial and the end behavior of the rational function.
v. Discuss how you can predict the end behavior of the graph of the rational function without rewriting the function.

$$g(x)=\frac{5x^2-8}{2x^2+1} \qquad G(x)=\frac{12x-5}{3x+2}$$

$$h(x)=\frac{3x^3-24}{x^2+1} \qquad H(x)=\frac{x^3-2}{x+1}$$

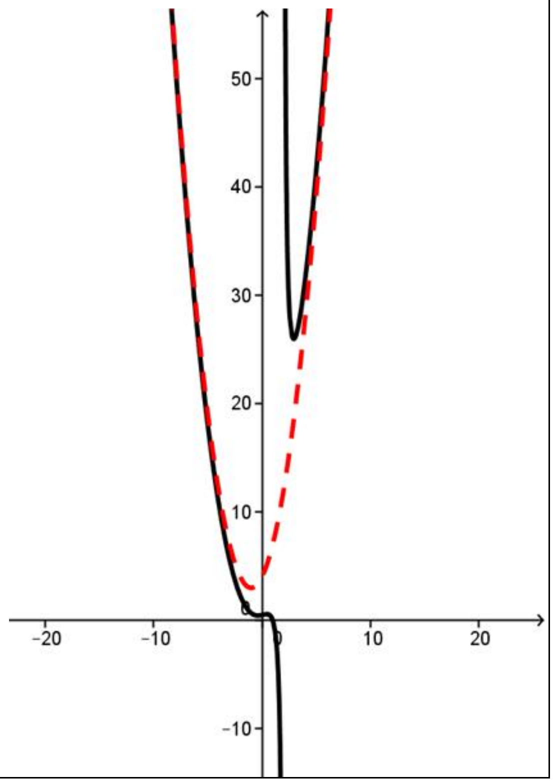
Prior Knowledge Needed:
Students need a basic understanding of the relationship between power functions and the end behavior of polynomials. Students need to know how to divide one polynomial by another polynomial to get a quotient consisting of a polynomial portion and a portion that is a rational function with the degree of the numerator less than the degree of the denominator. Students need to have a basic understanding of using a graphing calculator to graph polynomials and rational functions. Students may need to change the window settings in order to make the connections between the end behaviors of the graphs.
Prerequisite Common Core State Standards for Mathematical Content that support this example
Interpret the structure of expressions Write expressions in equivalent forms to solve problems Rewrite rational expressions Interpret functions that arise in applications in terms of the context Analyze functions using different representations *The complete Common Core State Standards for High School Mathematics can be found at http://www.corestandards.org/math

Solutions

Function f(x):

(i): $f(x)=\dfrac{x^3-1}{x-2}=x^2+2x+4+\dfrac{7}{x-2}$

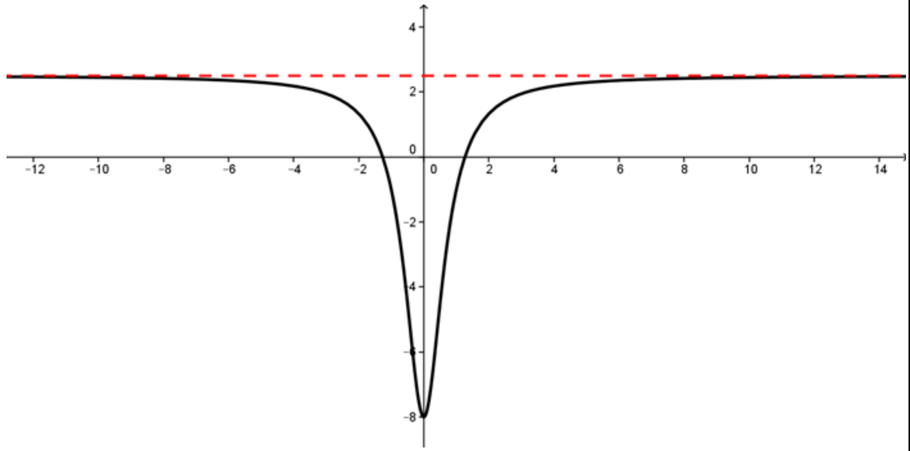
(ii) and (iii): The function (in black) and the polynomial part (in red) are graphed to the right.



(iv): The graph of the quadratic function closely follows the end behavior of the graph of the rational function.

Function g(x):

(i): $g(x)=\dfrac{5x^2-8}{2x^2+1}=\dfrac{5}{2}-\dfrac{\frac{21}{2}}{2x^2+1}$

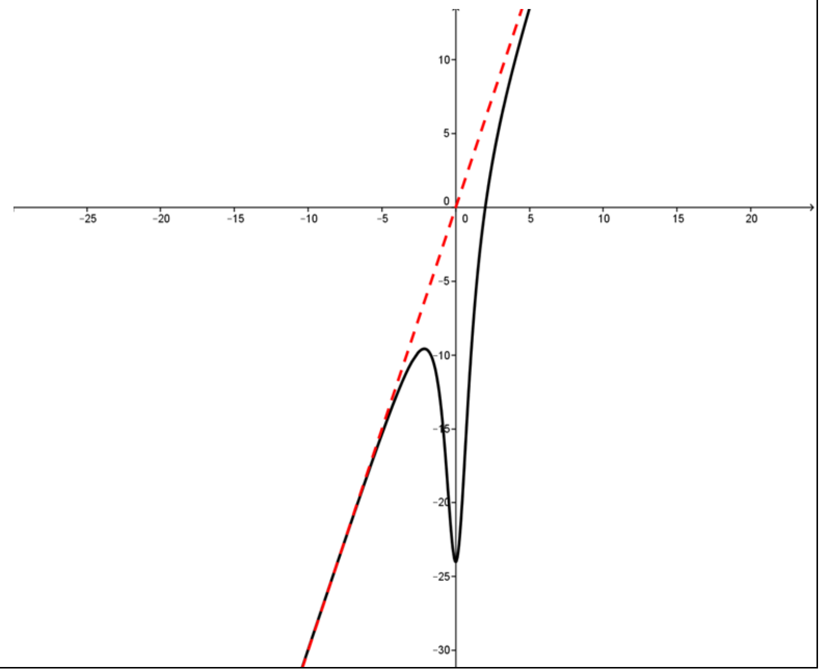


(ii) and (iii): The function (in black) and the polynomial part (in red) are graphed to the right.

(iv): The graph of the constant function closely follows the end behavior of the graph of the rational function.

Function h(x):

(i): $h(x)=\dfrac{3x^3-24}{x^2+1}=3x-\dfrac{3x+24}{x^2+1}$



(ii) and (iii): The function (in black) and the polynomial part (in red) are graphed to the right.

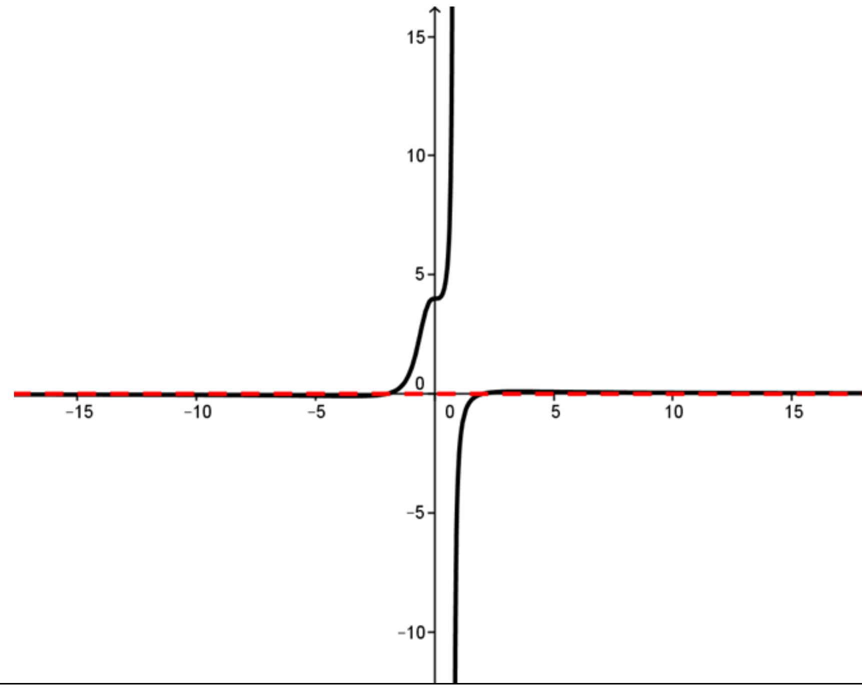
(iv): The graph of the linear function closely follows the end behavior of the graph of the rational function.

Function F(x):

(i): $F(x)=\dfrac{x^2-4}{2x^3-1}=0+\dfrac{x^2-4}{2x^3-1}$

(ii) and (iii): The function (in black) and the polynomial part (in red) are graphed to the right.

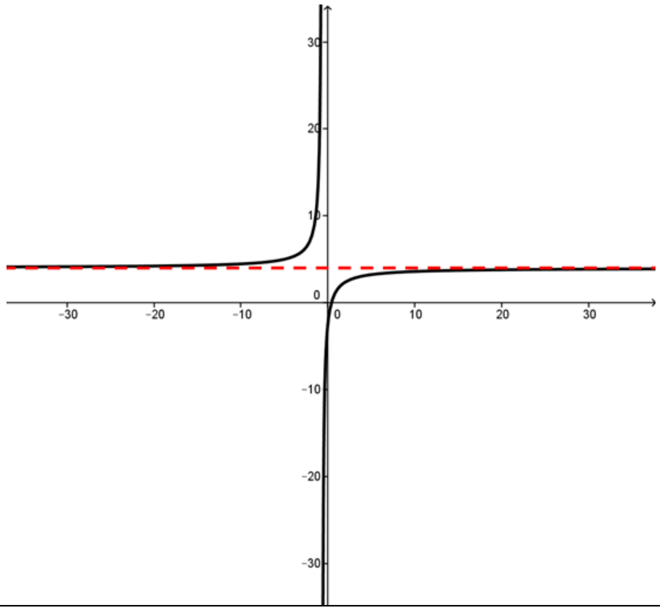
(iv): The graph of the zero function closely follows the end behavior of the graph of the rational function.



Function G(x):

(i): $G(x)=\dfrac{12x-5}{3x+2}=4-\dfrac{13}{3x+2}$

(ii) and (iii): The function (in black) and the polynomial part (in red) are graphed to the right.



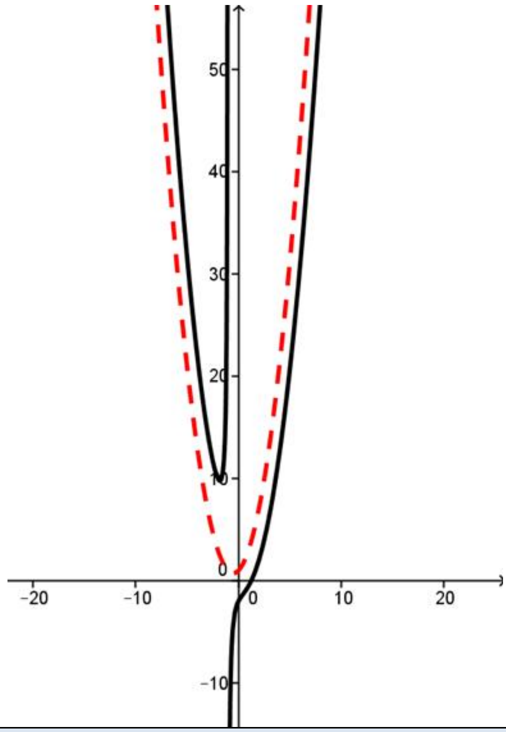
(iv): The graph of the constant function closely follows the end behavior of the graph of the rational function.

Function H(x):

(i): $H(x)=\dfrac{x^3-2}{x+1}=x^2+x+1-\dfrac{1}{x-1}$

(ii) and (iii): The function (in black) and the polynomial part (in red) are graphed to the right.

(iv): The graph of the quadratic function closely follows the end behavior of the graph of the rational function.



Tasks for Student Work

Predict the end behavior of each of these functions and justify your answer.

$f(x)=\dfrac{8x+3}{2x-1}$ $\qquad g(x)=\dfrac{x}{x^2-9}$ $\qquad h(x)=\dfrac{x^2+x+3}{x+1}$ $\qquad k(x)=\dfrac{5-x}{x+2}$

Examining the End Behavior of Rational Functions
Classroom Task

For each of the following rational functions:

i. Rewrite the rational function as the sum of a polynomial and a rational function whose numerator has a smaller degree than its denominator. (Remember, a polynomial can be a constant.)
ii. Graph the original rational function using your graphing calculator.
iii. Graph the polynomial portion of your rewritten form of the original rational function.
iv. Discuss any relationships you see between the end behavior of the polynomial and the end behavior of the rational function.
v. Discuss how you can predict the end behavior of the graph of the rational function without rewriting the function.

$$f(x)=\frac{x^3-1}{x-2}$$

$$F(x)=\frac{x^2-4}{2x^3-1}$$

$$g(x)=\frac{5x^2-8}{2x^2+1}$$

$$G(x)=\frac{12x-5}{3x+2}$$

$$h(x)=\frac{3x^3-24}{x^2+1}$$

$$H(x)=\frac{x^3-2}{x+1}$$

Examining the End Behavior of Rational Functions
Student Work

Predict the end behavior of each of these functions and justify your answer.

$$f(x) = \frac{8x+3}{2x-1}$$

$$g(x) = \frac{x}{x^2-9}$$

$$h(x) = \frac{x^2+x+3}{x+1}$$

$$k(x) = \frac{5-x}{x+2}$$

Supplementary Task: The Importance of Horizontal Asymptotes

This task is designed to support students in learning about the importance of studying horizontal asymptotes by providing a context in which the horizontal asymptote provides needed information. There are two examples in this task. One focuses on the horizontal asymptote for a rational function and one focuses on the horizontal asymptotes for a logistic function. Since the two examples are independent of each other, instructors may choose to use one or both examples as appropriate within their own existing coursework. Instructors are encouraged to adapt the examples to fit their style of teaching and their content expectations.

Example 2 includes both a table of values and a logistic model based on those values (values in the model have been rounded). An instructor may choose to provide both the table of values and the model, just the model, or just the table of values when presenting the example to students.

These examples support the objective:

> Use functions to model and solve problems using tables, graphs, and algebraic properties. Interpret constants, coefficients, and bases in the context of the problem.

in the Core to College Master Syllabus for Core-Aligned College Algebra.

Example 1: Rational Functions

A drug is injected into a patient and the concentration of the drug in the bloodstream is monitored. The drug's concentration, C(t), in milligrams per liter, after t hours is modeled by:

$$C(t) = \frac{11t}{2t^2 + 1}$$

a) Graph the function. Explain any restrictions on the domain within the context of the problem.
b) Use the graph to determine what happens as time passes and justify your response.
c) Researchers are using a new drug to treat the same condition as the drug described above. The concentration N(t) of the new drug in the bloodstream after t hours is given by:

$$N(t) = \frac{t}{3t^2 - 24t + 49}$$

where N(t) is measured in milligrams per liter. Graph this function and compare the graph of N(t) to the graph of C(t). Which drug do you think would be more effective? Explain why you chose as you did. Use the graphs to justify your response.

Example 2: Logistic Function (Exponential Functions)

The decennial population of Shelby County, according to the U.S. Census Bureau, is recorded in the table below.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
Population	153,557	191,439	223,216	306,482	358,250	482,393	627,019	722,014	777,113	826,330	897,472	927,640

The population follows a logistic model, given below, where P(t) represents the population t years after 1900.

$$P(t) = \frac{1030437}{1+(7.43\, e^{(-0.039\, t)})}$$

a) Graph the function.
b) If the population trend described by the function continues, what will happen to the population of Shelby County as time passes? Use the graph to justify your answer.

Prior Knowledge Needed:

For example 1, students will need a basic familiarity with rational functions, including finding horizontal asymptotes.

For example 2, students will need a basic familiarity with properties of exponents and exponential functions. The fact that e has a negative exponent in the denominator may cause difficulty for some students.

Prerequisite Common Core State Standards for Mathematical Content that support these examples

Understand the concept of a function and use function notation

Interpret functions that arise in applications in terms of the context

Analyze functions using different representations

Interpret expressions for functions in terms of the situation they model

*The complete Common Core State Standards for High School Mathematics can be found at http://www.corestandards.org/math

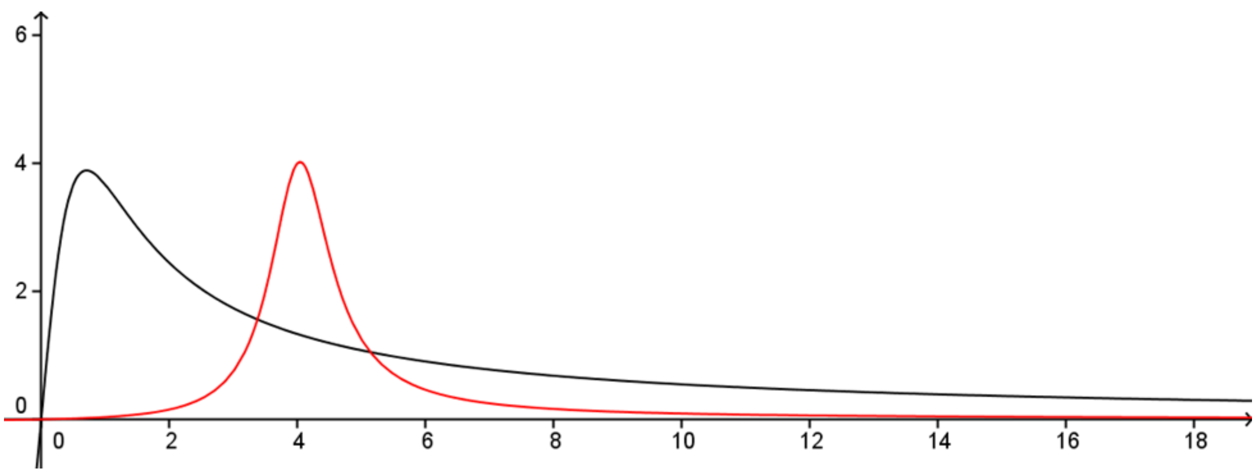
Solutions

Problem 1:

Students may approach this problem in several ways. First, they may substitute larger and larger values of t to discover what happens to C(t) as t increases. Second, they may use a graphing calculator to graph the function and determine the end behavior of the graph. Finally, they may use a theorem and compare the degree of the numerator to the degree of the denominator to determine the presence of a horizontal asymptote. In any case, students should realize that as t increases, the concentration of the drug in the bloodstream decreases and approaches 0, and students should be able to justify their conclusion using the graph of the function. (This is to be expected, as the drug will eventually "wear off.")

The graph below shows the graph of the first drug (in black) and the "new" drug (in red). As can be seen from the graph, the concentration of the first drug peaks more quickly than the concentration of the second drug. The effects from the second drug do not last as long as the effects of the first drug.

The "effectiveness" of the drug is subjective. If, for example, these drugs are painkillers, the first drug would be more effective—the pain would be lessened earlier and the effects of the painkiller would last longer. Suppose, however, that the drugs were chemotherapy drugs that have drastic side effects as long as the drug remains in the patient's system. In this case, a patient might choose the second drug since the concentration in the bloodstream decreases so quickly.



Problem 2:

As described in problem 1, students may solve this problem either by substituting larger and larger values of t or they may use a graphing calculator to graph the function and determine the end behavior. A third option would be to consider properties of the exponential function:

> As t gets large, the graph of $y = e^{-t}$ will approach 0 (look at the graph's right-hand side). That means that the larger the value of t is, the closer to 0 the quantity $7.43\ e^{(-0.039\ t)}$ will be. Thus, as t gets large, the denominator gets closer to 1, so the value of P(t) approaches 1030437.

In terms of the problem, this means that the population of Shelby County is expected to "top out" at 1,030,437 if the trend established by the previous censuses continues.

Tasks for Student Work

A biology class performs an experiment comparing the quantity of food consumed by a certain kind of moth with the quantity supplied. The model for the experimental data is:

$$C(x) = \frac{1.648x - 0.002}{5.954x + 1}, \qquad x > 0$$

where x represents the quantity (in milligrams) of food supplied and y is the quantity (in milligrams) of food consumed. At what level of consumption will the moth become satiated?

An organization dedicated to preserving endangered species released 100 prairie chickens in a game preserve. The preserve has a carrying capacity of 1000 prairie chickens. The growth of the flock is modeled by: $p(t) = \dfrac{1000}{1 + 8.75e^{-0.1765t}}$, where t is measured in months.

a) Estimate the population after 6 months.
b) How long will it take the population to reach 500?
c) Determine the horizontal asymptotes, and interpret the horizontal asymptotes within the context of the problem.

The Importance of Horizontal Asymptotes:
Example 1

A drug is injected into a patient and the concentration of the drug in the bloodstream is monitored. The drug's concentration, C(t), in milligrams per liter, after t hours is modeled by:

$$C(t) = \frac{11t}{2t^2 + 1}$$

a) Graph the function. Explain any restrictions on the domain within the context of the problem.
b) Use the graph to determine what happens as time passes and justify your response.
c) Researchers are using a new drug to treat the same condition as the drug described above. The concentration N(t) of the new drug in the bloodstream after t hours is given by:

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The Importance of Horizontal Asymptotes:
Example 2

The decennial population of Shelby County, according to the U.S. Census Bureau, is recorded in the table below.

Year	1900	1910	1920	1930	1940	1950
Population	153,557	191,439	223,216	306,482	358,250	482,393

table continued:

Year	1960	1970	1980	1990	2000	2010
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$$P(t) = \frac{1030437}{1+(7.43\,e^{(-0.039\,t)})}$$

a) Graph the function.
b) If the population trend described by the function continues, what will happen to the population of Shelby County as time passes? Use the graph to justify your answer.

The Importance of Horizontal Asymptotes:
Student Task

A biology class performs an experiment comparing the quantity of food consumed by a certain kind of moth with the quantity supplied. The model for the experimental data is:

$$C(x) = \frac{1.648x - 0.002}{5.954x + 1}, \qquad x > 0$$

where x represents the quantity (in milligrams) of food supplied and y is the quantity (in milligrams) of food consumed. At what level of consumption will the moth become satiated?

The Importance of Horizontal Asymptotes:
Student Task

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a) Estimate the population after 6 months.
b) How long will it take the population to reach 500?
c) Determine the horizontal asymptotes, and interpret the horizontal asymptotes within the context of the problem.

Supplementary Task: Presenting Data (What's the "Big Whoop"?)

This task is designed to illustrate the strengths and weaknesses of data presented in a table, as a graph, or as a function. The task can be used at any point in a college algebra course when students are using data to develop a model for an application.

These examples support the objective:

Analyze data and apply concepts to solve problems.

in the Core to College Master Syllabus for Core-Aligned College Algebra.

Example 1: Whooping Cranes Population

Whooping cranes are the tallest birds in North America. The whooping crane nearly vanished in the mid-20th century, with a count of only 16 birds in 1941. Captive breeding programs and reintroduction efforts have increased the number of wild birds to over 200, with roughly the same number living in captivity. The last remaining natural migratory flock of whooping cranes is called the Western flock. (In 2001, experts began to introduce an Eastern flock as part of reintroduction efforts.) The population of the whooping crane Western flock from 1940 until 2010 is given in the table below.

Year	1940	1950	1960	1970	1980	1990	2000	2010
# Cranes	22	34	33	56	76	146	177	281

a) What kinds of information does this table of values give you?

b) Graph the data given in the table. Does the graph give you more or different information about general trends in the data? If so, what kind of information does the graph give you that the table does not?

c) Find a function to model the data given in the table. Graph your function on the graph from part (b). How well does the graph of your function fit the data from the table? What information does a function allow you to infer that would not be apparent from a table or a graph?

d) Using the function, predict the population of the whooping crane Western flock in 2025. Does this value seem reasonable?

Resource: www.learner.org/jnorth/tm/crane/Population.html

Example 2: Pertussis (Whooping Cough) Cases

Pertussis is a highly contagious respiratory disease known for uncontrollable, violent coughing. Pertussis most commonly affects infants and young children. Diagnosed pertussis cases are reported by states to the Centers for Disease Control and Prevention

(CDC). The number of reported pertussis cases at five-year intervals from 1925 until 2010 are included in the table below.

Year	1925	1930	1935	1940	1945	1950	1955	1960	1965
# Reported Cases	152003	166914	180518	183866	133792	120718	62786	14809	6799

(table continued)

Year	1970	1975	1980	1985	1990	1995	2000	2005	2010
# Reported Cases	4249	1738	1730	3589	4570	5137	7867	25619	27550

a) What kinds of information does this table of values give you?

b) Graph the data given in the table. Does the graph give you more or different information about general trends in the data? If so, what kind of information does the graph give you that the table does not?

c) Find a function to model the data given in the table. Graph your function on the graph from part (b). How well does the graph of your function fit the data from the table? What information does a function allow you to infer that would not be as apparent from a table or a graph?

d) Use your function to predict the number of reported pertussis cases in 2025. Does this value seem reasonable?

e) The number of reported pertussis cases began to rise between 1980 and 1985, and there was a significant increase between 2000 and 2005. What do you think caused this increase?

Resource: Centers for Disease Control and Prevention, www.cdc.gov/pertussis/surv-reporting/cases-by-year.html (Note: This site includes data for all years 1922-2011.)

Prior Knowledge Needed

Students should have a basic working knowledge of using data from a table to create a scatterplot (most likely using technology). Students should also know how to use technology to create a regression function (linear, quadratic, cubic, quartic, exponential, etc.).

It is also helpful if the technology used provides a value of the coefficient of determination (r^2). If the technology provides a value for r^2, it will be useful for students to know that higher r^2 values imply better fits between the model and the data.

Prerequisite Common Core State Standards for Mathematical Content that support these examples

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice.

Interpret functions that arise in applications in terms of the context

Analyze functions using different representations

Build a function that models a relationship between two quantities

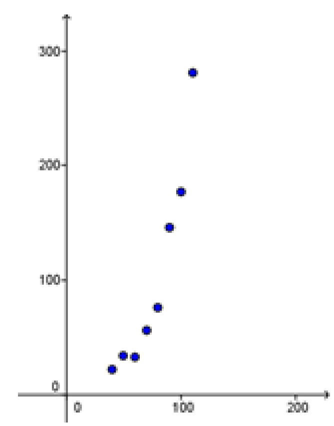
Construct and compare linear, quadratic, and exponential models and solve problems

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

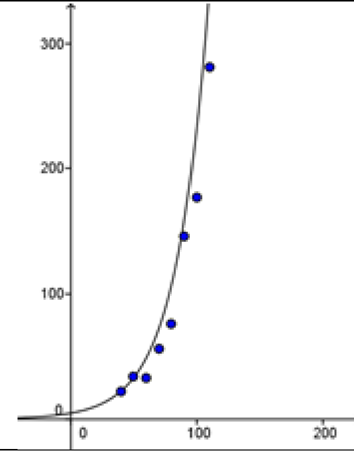
Solutions

Example 1, part (a): A table of values will give specific information at specific points in the problem. Sometimes it is possible to identify overall trends (increasing, decreasing, constant, for example), but it is usually difficult to determine other information such as rates of increase or decrease or predicted values for points not given in the table.

Example 1, part (b): The graph appears to the right. Note that the graph indicates a very steep rise in the population in this case, so there is apparently a high rate of increase in the population. The shape of the graph also suggests an exponential model.



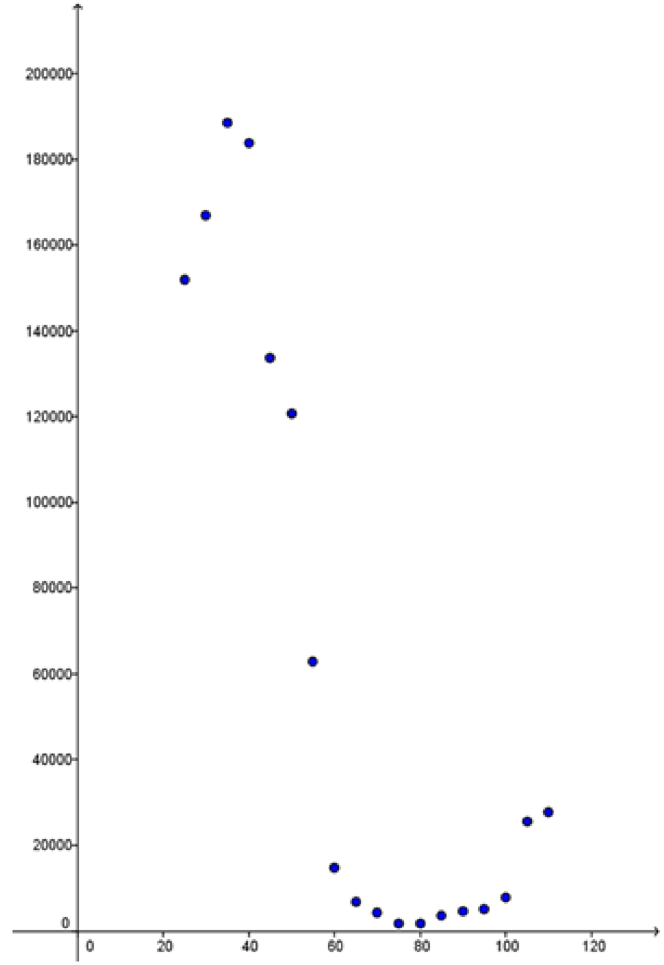
Example 1, part (c): Based on the shape of the graph, an exponential function should be a good fit for the model. Using exponential regression on the calculator, we can construct an exponential model: $P(t) = 4.62(1.04)^t$, where P(t) represents the population t years after 1900. (Note that values from the regression are rounded to two decimal places.) The graph of the model appears to the right. The function provides an easy way to estimate values of the population at points not originally given in the table.



Example 1, part (d): Using P(t) (with the rounded values) from part (c) and using t = 125, we would predict the population in 2025 to be approximately 622 whooping cranes in this population. This is reasonable given that the year of the prediction is 15 years after the last value given in the table.

Example 2, part (a): A table of values will give specific information at specific points in the problem. Sometimes it is possible to identify overall trends (increasing, decreasing, constant, for example), but it is usually difficult to determine other information such as rates of increase or decrease or predicted values for points not given in the table.

Example 2, part (b): The graph is given on the right. From the graph, it is clear that there is a rise in the number of cases, followed by a sharp decline, and then a smaller rise. This suggests that perhaps a higher-degree polynomial model might be a better fit than an exponential, linear, quadratic, logarithmic, or logistic model.

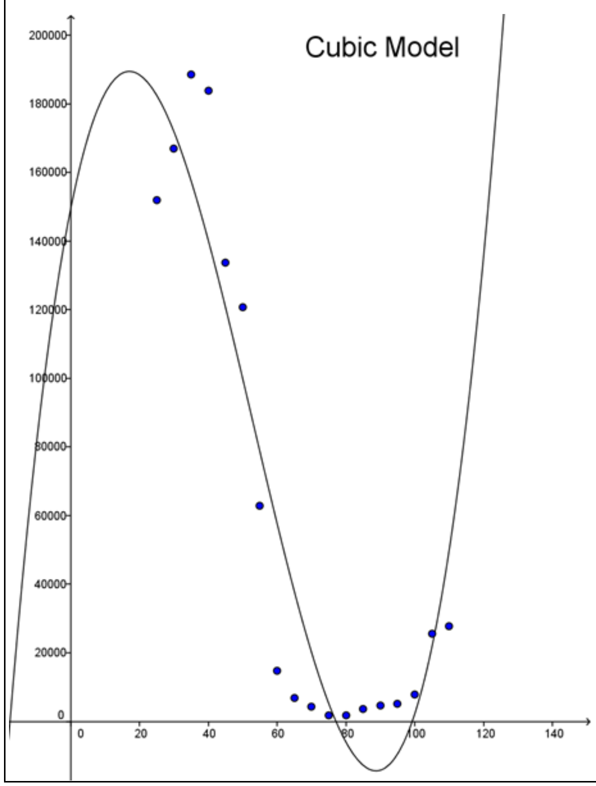


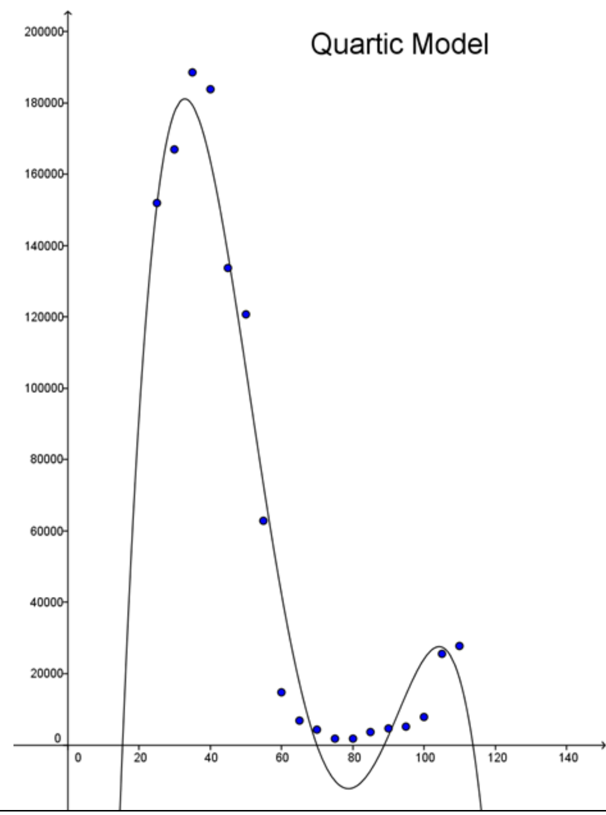
Example 2, part (c): Both a cubic and a quartic model are given. The data were input into a graphing utility using, as the t-value, the number of years since 1900. Values given by the graphing utility were rounded to four decimal places due to the sizes of the coefficients and the spread of points in the graph. Keeping as many decimal places as possible will result in closer fits to the data.

The quartic model is a slightly better fit than the cubic model. The function provides an easy way to estimate values of the population at points not originally given in the table.

Cubic model: $N(t) = 1.1075t^3 - 175.8012t^2 + 5034.9750t + 149256.0743$

Quartic model: $N(t) = -0.0619t^4 + 17.8206t^3 - 1761.9939t^2 + 66873.2765t - 673861.8579$





Example 2, part (d): Values are calculated using the N(t) functions given in part (c). The value of t is 125 (years since 1900). Cubic model: N (125) = 194820 (rounded to the nearest integer) – This does not seem reasonable as this value is higher than any of the numbers of reported pertussis cases given in the table. We would expect that modern medicine would be able to cope with the increase in these numbers and would be able to stop the rapid increase. Quartic model: N(125) = -152302 (rounded to the nearest integer) – This does not seem reasonable since we cannot have a negative number of reported cases of pertussis.
Example 2, part (e): According to the Center for Disease Control, reported cases of whooping cough vary from year to year and tend to peak every 3-5 years. This pattern is not completely understood. Many factors contribute to the number of reported pertussis cases, including improvements in diagnosis and the reporting of cases in adolescents and adults.

Presenting Data
Example 1: Whooping Cranes

Whooping cranes are the tallest birds in North America. The whooping crane nearly vanished in the mid-20th century, with a count of only 16 birds in 1941. Captive breeding programs and reintroduction efforts have increased the number of wild birds to over 200, with roughly the same number living in captivity. The last remaining natural migratory flock of whooping cranes is called the Western flock. (In 2001, experts began to introduce an Eastern flock as part of reintroduction efforts.) The population of the whooping crane Western flock from 1940 until 2010 is given in the table below.

Year	1940	1950	1960	1970	1980	1990	2000	2010
# Cranes	22	34	33	56	76	146	177	281

a) What kinds of information does this table of values give you?

b) Graph the data given in the table. Does the graph give you more or different information about general trends in the data? If so, what kind of information does the graph give you that the table does not?

c) Find a function to model the data given in the table. Graph your function on the graph from part (b). How well does the graph of your function fit the data from the table? What information does a function allow you to infer that would not be apparent from a table or a graph?

d) Using the function, predict the population of the whooping crane Western flock in 2025. Does this value seem reasonable?

Resource: www.learner.org/jnorth/tm/crane/Population.html

Presenting Data
Example 2: Pertussis (Whooping Cough)

Pertussis is a highly contagious respiratory disease known for uncontrollable, violent coughing. Pertussis most commonly affects infants and young children. Diagnosed pertussis cases are reported by states to the Centers for Disease Control and Prevention (CDC). The number of reported pertussis cases at five-year intervals from 1925 until 2010 are included in the table below.

Year	1925	1930	1935	1940	1945	1950	1955	1960	1965
# Reported Cases	152003	166914	180518	183866	133792	120718	62786	14809	6799

(table continued)

Year	1970	1975	1980	1985	1990	1995	2000	2005	2010
# Reported Cases	4249	1738	1730	3589	4570	5137	7867	25619	27550

a) What kinds of information does this table of values give you?

b) Graph the data given in the table. Does the graph give you more or different information about general trends in the data? If so, what kind of information does the graph give you that the table does not?

c) Find a function to model the data given in the table. Graph your function on the graph from part (b). How well does the graph of your function fit the data from the table? What information does a function allow you to infer that would not be as apparent from a table or a graph?

d) Use your function to predict the number of reported pertussis cases in 2025. Does this value seem reasonable?

e) The number of reported pertussis cases began to rise between 1980 and 1985, and there was a significant increase between 2000 and 2005. What do you think caused this increase?

Resource: Centers for Disease Control and Prevention, www.cdc.gov/pertussis/surv-reporting/cases-by-year.html (Note: This site includes data for all years 1922-2011.)

Supplementary Task: Applications of Systems of Linear Equations with Infinitely Many Solutions

Students often fail to understand the significance of "infinitely many solutions" for a system of linear equations. In most cases, students may be able to recognize that such a system has infinitely many solutions, but do not know how to describe those solutions or why anyone would want to know a general form for those solutions. This task is designed to provide a context for a system of linear equations in which the "infinitely many solutions" case is vital to the context of the problem.

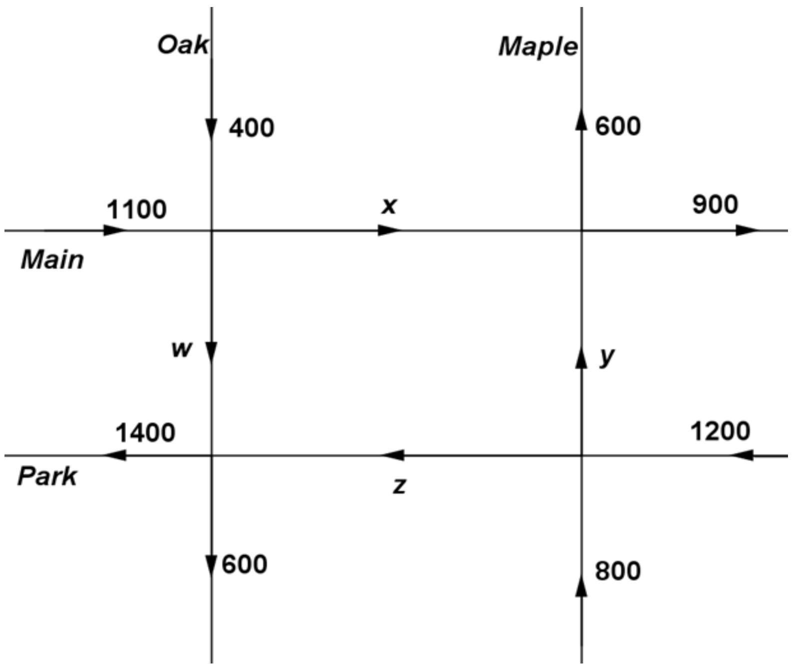
This examples supports the objective:

Use systems of two or more equations or inequalities to solve problems using tables, graphs, and algebraic properties.
Interpret intersections/regions in the context of the problem.

in the Core to College Master Syllabus for Core-Aligned College Algebra.

Example: Traffic Flow

The figure below shows the flow of traffic in a city during the rush hours on a typical weekday. The arrows indicate the direction of the flow of traffic on each one-way road. The average number of vehicles per hour entering and leaving each intersection is indicated beside each road. Oak and Maple streets can each handle up to 1000 cars per hour without congestion. Main and Park streets can each handle up to 1600 cars per hour without congestion. Traffic lights installed at each of the four intersections control the flow of traffic.



a) Explicitly state a general expression involving the rates of flow (x, y, z, and w) and suggest two different possible flow patterns that will ensure no traffic congestion.

b) Suppose there is a water main break on Oak Street between Main and Park, and the traffic along Oak is narrowed to one lane. By narrowing to one lane, the maximum number of cars per hours on Oak Street is reduced to 600. Find two different possible flow patterns that will ensure no traffic congestion with this restriction.

Prior Knowledge Needed:

Students will need to know at least one method for solving a system of linear equations. Students will need some background knowledge regarding solving systems of linear equations with infinitely many solutions.

Students may need assistance understanding the vocabulary associated with traffic flow patterns.

Prerequisite Common Core State Standards for Mathematical Content that support this example

Create equations that describe numbers or relationships

Solve systems of equations

*The complete Common Core State Standards for High School Mathematics can be found at http://www.corestandards.org/math

Solution

Setting up the equations:

Each intersection will give rise to a linear equation. The number of cars flowing in to an intersection must equal the number of cars flowing out of the intersection. We will call the intersection of Oak and Main intersection "A", then move in a counterclockwise direction to name intersections B, C, and D.

Intersection	Flow Into Intersection	Flow Out of Intersection
A	1100 + 400	x + w
B	w + z	1400 + 600
C	800 + 1200	y + z
D	x + y	600 + 900

Thus, our system of equations is:

$$x + w = 1500$$
$$w + z = 2000$$
$$y + z = 2000$$
$$x + y = 1500$$

Solving the system:

Students may use the methods of elimination or substitution to solve the system by hand. If students are familiar with matrices, they may put the augmented matrix associated with the system in reduced row echelon form either by hand or using technology.

This system will reduce to an equivalent system of three equations in four variables, so we must choose an arbitrary value for one of the variables. If we let w be the arbitrary variable, then the solution to our system is:

$$x = 1500 - w$$
$$y = w$$
$$z = 2000 - w$$

Limitations on the values of the variables:

To write a solution to the problem, we must consider whether there are any constraints on the variables. In the problem, we are told that Oak and Maple streets can each handle up to 1000 cars, and Main and Park streets can each handle up to 1600 cars. Thus, in our solutions, we need to take these constraints into account.

The variable w describes the flow of traffic on Oak Street, so we know that our values of w cannot exceed 1000. Similarly, neither x nor z can exceed 1600, and y cannot exceed 1000. Since all four variables represent the number of cars using the street, we know that all four variables must be non-negative.

To describe the flow patterns, we will need to choose a value for w and then calculate the values of x, y, and z based on the value chosen for w. Note that since $z = 2000 - w$ and since z cannot exceed 1600, we know that w must be at least 400 in order to determine a valid value for z.

This will also have consequences for the possible values of x and y.

Solution to part (a):

The general expression for the rates of flow are given as:

$$x = 1500 - w$$
$$y = w$$
$$z = 2000 - w$$

where $400 \le w \le 1000$.

Flow patterns will depend on the number chosen for w. For example, if a student chooses $w = 500$, then we know that $x = 1000$, $y = 500$, and $z = 1500$.

Solution to part (b):

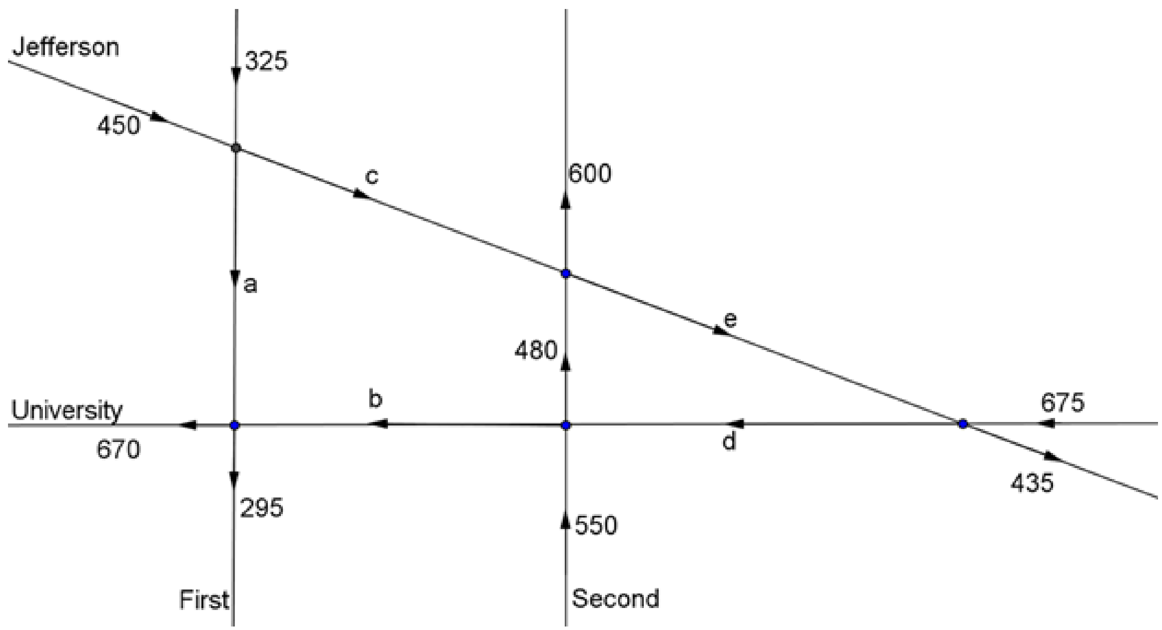
Reducing the maximum number of cars on Oak Street between Main and Park to 600 cars per hour will affect our possible solutions, but will not affect the set-up of the system or the general solution.

Since the maximum number of cars on Oak Street is now 600, our possible values for w are now limited to $400 \le w \le 600$. Thus, a sample flow pattern using, say, $w = 450$ would be:

$w = 450$, $x = 1150$, $y = 450$, and $z = 1550$.

Task for Student Work

The figure below shows the flow of traffic in a city during the rush hours on a typical weekday. The arrows indicate the direction of the flow of traffic on each one-way road. The average number of vehicles per hour entering and leaving each intersection is indicated beside each road. Jefferson and University streets can each handle up to 800 cars per hour without congestion. First and Second streets can each handle up to 750 cars per hour without congestion. Traffic lights installed at each of the five intersections control the flow of traffic.



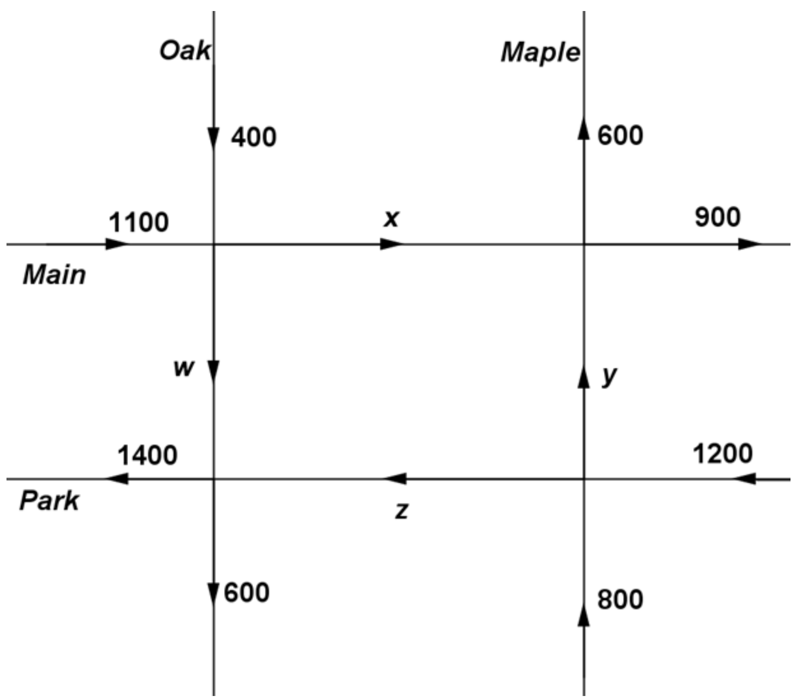
Explicitly state a general expression involving the rates of flow (a, b, c, d, and e) and suggest two different possible flow patterns that will ensure no traffic congestion.

Applications of Linear Equations
Example: Traffic Flow

The figure below shows the flow of traffic in a city during the rush hours on a typical weekday. The arrows indicate the direction of the flow of traffic on each one-way road. The average number of vehicles per hour entering and leaving each intersection is indicated beside each road. Oak and Maple streets can each handle up to 1000 cars per hour without congestion. Main and Park streets can each handle up to 1600 cars per hour without congestion. Traffic lights installed at each of the four intersections control the flow of traffic.

a) Explicitly state a general expression involving the rates of flow (x, y, z, and w) and suggest two different possible flow patterns that will ensure no traffic congestion.

b) Suppose there is a water main break on Oak Street between Main and Park, and the traffic along Oak is narrowed to one lane. By narrowing to one lane, the maximum number of cars per hours on Oak Street is reduced to 600. Find two different possible flow patterns that will ensure no traffic congestion with this restriction.



Applications of Linear Equations
Student Work

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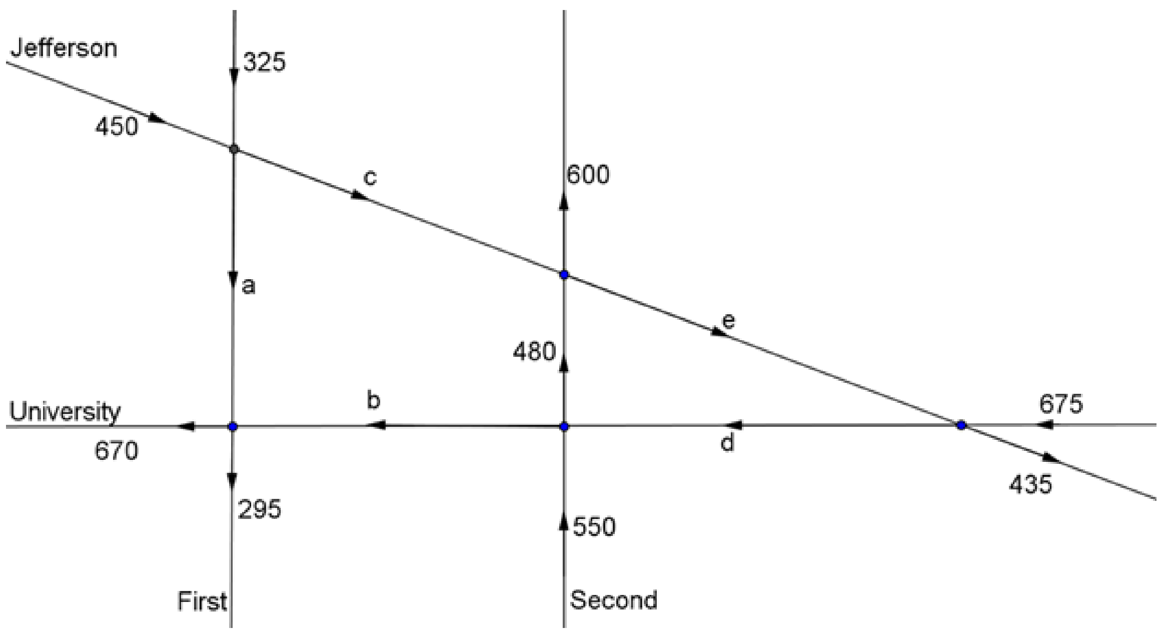


Explicitly state a general expression involving the rates of flow (a, b, c, d, and e) and suggest two different possible flow patterns that will ensure no traffic congestion.

Supplementary Task: Creating Models for Data

This task is designed to be a collection of data sets that can be used as classroom examples or as tasks for student work. Not all data sets provide clearly-determined solutions.

Culminating Project: Creating Models for Data

Data collected for various situations is presented below. For each set of data, students should be asked to:

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Data Set 1: Diameter of Sand Granules vs. Slope of the Beach

For naturally occurring ocean beaches, the data below were collected. Find a function to model the relationship between the median diameter of granules of sand (in millimeters) and the gradient of the slope of the beach (in degrees).

Diameter	0.17	0.19	0.22	0.235	0.235	0.3	0.35	0.42	0.85
Gradient	0.63	0.7	0.82	0.88	1.15	1.5	4.4	7.3	11.3

Resource: *Physical Geography*, by A. M. King, Oxford Press, England

Data Set 2: Cricket Chirps

The following data relate the number of chirps per second of the striped ground cricket and the temperature in degrees Fahrenheit. Find a function to model the relationship between these two quantities.

Chirps/sec	20.0	16.0	19.8	18.4	17.1	15.5	14.7	17.1	15.4	16.2	15.0	17.2	16.0	17.0
Temperature	88.6	71.6	93.3	84.3	80.6	75.2	69.7	82	69.4	83.3	79.6	82.6	80.6	83.5

Resource: *The Song of Insects*, by Dr. G. W. Pierce, Harvard College Press

Data Set 3: Ground Water Survey

The following data were collected from a random sample of wells in Northwest Texas. The data relate the bicarbonate (in parts per million) of the well water with the pH of the well water. Use the data to develop a model for the relationship between these two quantities.

Bicarbonate	157	174	175	188	171	143	217	190	142	190	215	199	262	105
pH	7.6	7.1	8.2	7.5	7.4	7.8	7.3	8.0	7.1	7.5	8.1	7.0	7.3	7.8

Resource: Union Carbide Technical Report K/UR-1

Data Set 4: Prehistoric Pueblos

The data below relate the estimated year of initial occupation with the estimated year of the end of occupation of prehistoric pueblos in a random sample of such pueblos in Utah, Arizona, and Nevada. Use the data to develop a model for the relationship between these two quantities.

Initial Year	900	700	1125	750	1250	1250	1175	1225	1180	1080	1080	1075	1090	1225	1200	1325
End Year	1250	1300	1175	1250	1300	1280	1225	1275	1250	1150	1275	1250	1135	1275	1285	1400

Resource: *Prehistoric Pueblo World*, by A. Adler, University of Arizona Press

Data Set 5: The Size of Alligators

Many wildlife populations are monitored by taking aerial photographs. For example, the length of an alligator can be estimated quite accurately from an aerial photograph. However, the alligator's weight is much more difficult to determine. The data below show the length (in inches) and weight (in pounds) of alligators captured in central Florida. Use the data to develop a model for the relationship between the length and weight of alligators in central Florida.

Length	58	61	63	68	69	72	72	74	74	76	78	82	85	86	86	86	88	89	90	90	94	94	114	128	147
Weight	28	44	33	39	36	38	61	54	51	42	57	80	84	83	80	90	70	84	106	102	110	130	197	366	640

Resource: Exploring Data website, http://curriculum.qed.qld.gov.au/kla/eda, Education Queensland, 1997

Data Set 6: Sunflower Height

The data below relates the height of single sunflower plant (in cm) to the number of days the plant has been growing. Use the data to develop a model for the relationship between the height and the number of days the plant has been growing.

Day	0	7	14	21	28	35	42	49	56	63	70	77	84
Height	0.00	17.93	36.36	67.76	98.10	131.00	169.50	205.50	228.30	247.10	250.50	253.80	254.50

Resource: Reed, H.S. and Holland, R.H. (1919), Growth of sunflower seeds; Proceedings of the National Academy of Sciences, volume 5, p. 140.

Data Set 7: Moving River Particles

Rivers and streams carry small solid particles of rock and mineral downhill, either suspended in the water column or bounced, rolled, or slid along the river bed. The data below show the speed (in m/sec) necessary to carry particles in suspension as it relates to the diameter of the particle (in mm). Use the data to develop a model for the relationship between the size of the particle in suspension and the speed necessary to move the particle.

Diameter	0.2	1.3	5	11	20	45	80	180
Speed	0.10	0.25	0.50	0.75	1.00	1.50	2.50	3.50

Resource: Nielsen, A. (1950) *Oikos,* 2, 176-96 as reported in Ecology for Environmental Sciences, Anderson J.M.

Data Set 8: World Records for Men's High Jump

The data below show the world record for the men's high jump (in meters) and the year in which the world record was set (between 1912 and 1993). Use the data to create a model for the relationship between the world record for the men's high jump and the year in which the record was set.

Year	1912	1914	1924	1933	1934	1936	1937	1941	1953	1956	1957	1960	1960
Record	2.00	2.01	2.03	2.04	2.06	2.07	2.09	2.11	2.12	2.15	2.16	2.17	2.18

(table continued)

Year	1960	1961	1961	1961	1962	1962	1963	1971	1973	1976	1976	1977	1978
Record	2.22	2.23	2.24	2.25	2.26	2.27	2.28	2.29	2.30	2.31	2.32	2.33	2.34

(table continued)

Year	1980	1980	1983	1983	1984	1985	1985	1987	1989	1993
Record	2.35	2.36	2.37	2.38	2.39	2.40	2.41	2.42	2.44	2.45

Data Set 9: World Records for Women's High Jump

The data below show the world record for the women's high jump (in meters) and the year in which the world record was set (between 1932 and 1987). Use the data to create a model for the relationship between the world record for the women's high jump and the year in which the record was set.

Year	1932	1939	1943	1951	1954	1956	1956	1956	1957	1958	1958	1958	1958
Record	1.65	1.66	1.71	1.72	1.73	1.74	1.75	1.76	1.77	1.78	1.80	1.81	1.82

(table continued)

Year	1958	1958	1960	1960	1961	1961	1961	1971	1972	1974	1976	1977	1977
Record	1.83	1.84	1.85	1.86	1.88	1.90	1.91	1.92	1.94	1.95	1.96	1.97	2.00

(table continued)

Year	1978	1982	1983	1983	1984	1984	1986	1987
Record	2.01	2.02	2.03	2.04	2.05	2.07	2.08	2.09

Data Set 10: Space Shuttle Ascent Altitude

On July 4, 2006, the Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121 to begin a rendezvous with the International Space Station. The data below show the altitude (in feet) of Discovery every 10 seconds from liftoff to the separation of the solid rocket boosters. Use the data to model the altitude of Discovery at time t.

Time	0	10	20	30	40	50	60	70	80	90	100	110	120
Altitude	7	938	4160	9872	17635	26969	37746	50548	66033	83966	103911	125512	147411

Resource: www.nasa.gov

Data Set 11: Space Shuttle Ascent Total Mass

On July 4, 2006, the Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121 to begin a rendezvous with the International Space Station. The data below show the total mass (in kg) of Discovery every 10 seconds from liftoff to the separation of the solid rocket boosters. Use the data to model the total mass of Discovery at time t.

Time	0	10	20	30	40	50	60	70	80	90	100
Total Mass	2,051,113	1,935,155	1,799,290	1,681,120	1,567,611	1,475,282	1,376,301	1,277,921	1,177,704	1,075,683	991,872

(table continued)

Time	110	120
Total Mass	913,254	880,377

Resource: www.nasa.gov

Prior Knowledge Needed:

Many of these data sets are available on the Data and Story Library (http://lib.stat.cmu.edu/DASL/). In some cases, only a partial data set is included here.

Students should have a basic working knowledge of using data from a table to create a scatterplot (most likely using technology). Students should also know how to use technology to create a regression function (linear, quadratic, cubic, quartic, exponential, etc.)

It is also helpful if the technology used provides a value of the coefficient of determination (r^2). If the technology provides a value for r^2, it will be useful for students to know that higher r^2 values imply better fits between the model and the data.

Common Core State Standards for Mathematical Content that support this example

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice.

Interpret functions that arise in applications in terms of the context

Analyze functions using different representations

Build a function that models a relationship between two quantities

Construct and compare linear, quadratic, and exponential models and solve problems

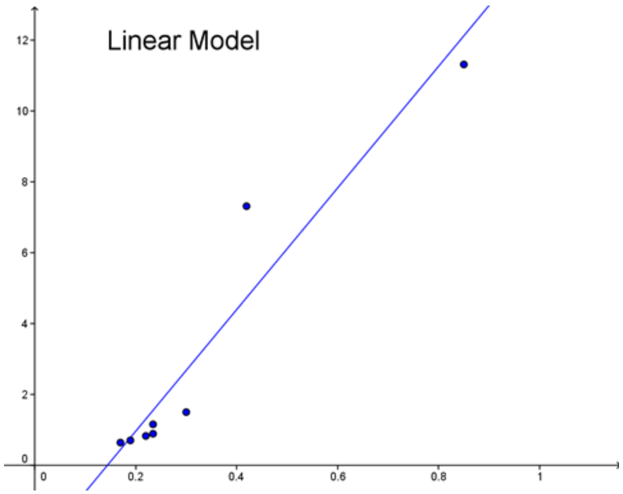
Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Solutions

A complete solution is provided for Data Set 1 as an example. For other data sets, the data plot and the recommended model(s) are provided.

Data Set 1:

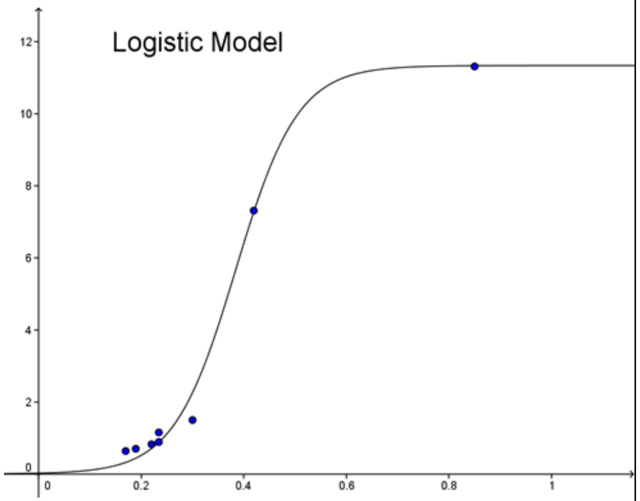
a) Based on the graph, the logistic model is the most likely best-fitting model. A linear model can be used as well. (Values are rounded to the nearest hundredth.)



Linear model: $G(d) = 17.16d - 2.48$, where d = diameter of the granules of sand in millimeters and G(d) represents the gradient of the slope of the beach in degrees. (Here, the correlation coefficient r is approximately 0.95, indicating that the model is a good fit for the data.)

Logistic model: $G(d) = \dfrac{11.33}{1 + 577.98e^{-16.54d}}$, where d = diameter of the granules of sand in millimeters and G(d) represents the gradient of the slope of the beach in degrees.

b) Graphs: The graphs of both models appear to the right.

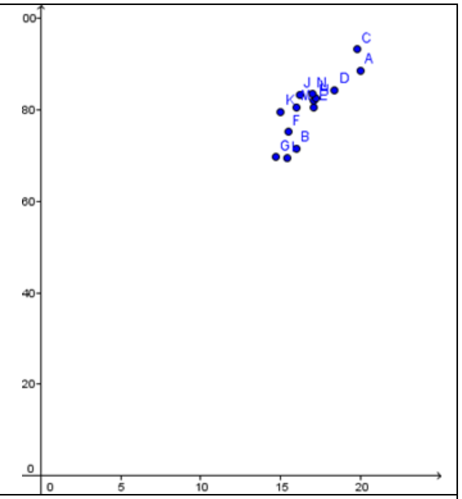


c) Limitations of the model: Both models can be used to predict values of the gradient based on values of the diameter of the granules of sand that lie between 0.17 mm (the smallest diameter given) and 0.85 mm (the largest diameter given). The models should not be used to predict values outside of this domain. Also note that the size of the granules of sand should not increase without bound; there should be a limiting value of the size of these granules. Similarly, there should be a limiting value of the gradient of the slope of the beach.

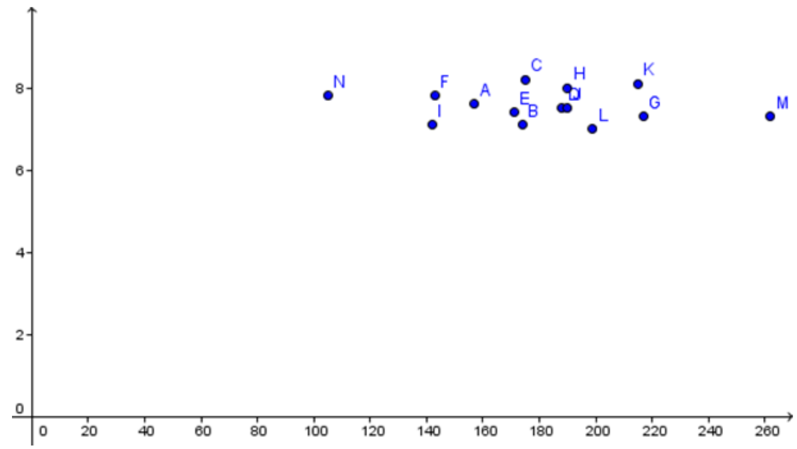
d) Sample Questions (other questions may be asked):

- For the linear model, what does the slope of the line represent? What does the y-intercept represent?
- Predict the gradient of the slope of the beach given a particular diameter not listed in the table.
- Predict the diameter of a granule of sand given a particular gradient of the slope of a beach not listed in the table.
- For the logistic model, what does the model suggest will happen to the gradient of the slope of a beach as the size of the granules of sand increases?

Data Set 2: This is a fairly common example of an application of linear regression. However, the data given can also be modeled using polynomials with approximately the same coefficient of determination as the linear model.

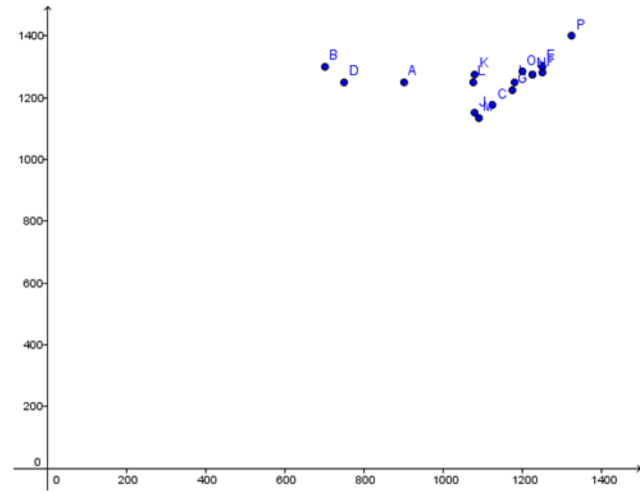


Data Set 3: This data is very scattered and does not fit the standard models very well. The data set is included for use as an example to illustrate that not all data sets can be modeled effectively.

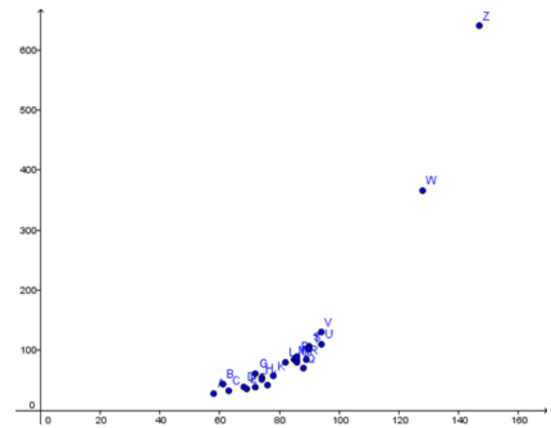


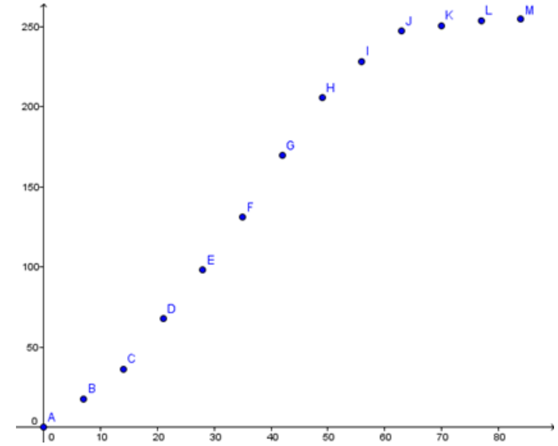
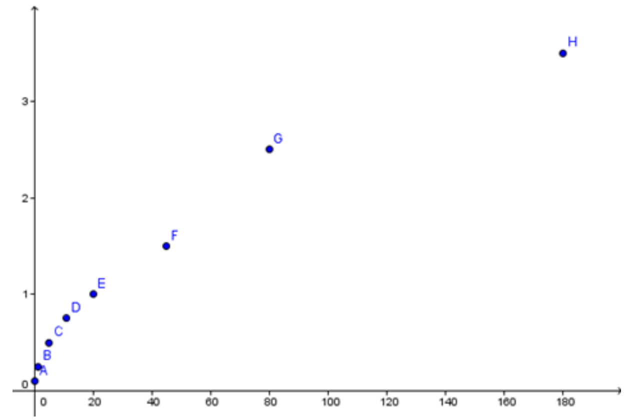
Data Set 4: This data set is not as scattered as data set 3, but there seems to be two “trends” in the graph of the data. Therefore, the standard models do not provide a close fit. A comparison of the models indicates that a cubic or quartic model fits most closely, with a quadratic model only a little less effective.

One way to improve the modeling process is to consider not the “initial year-ending year” data but to consider “initial year-*estimated length of occupation*” data that can be developed by finding the difference in the ending year and the initial year for each pueblo. This data gives rise to better-fitting models, with quadratic, cubic, and quartic models providing the best fits. Logarithmic and linear models also provide good fits, but these are not as close as the polynomial models.

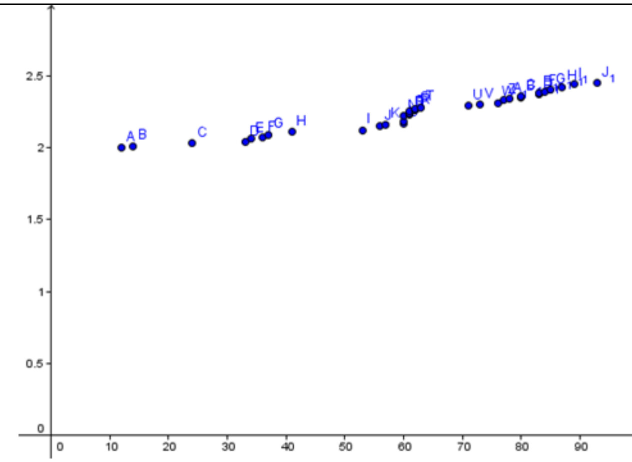


Data Set 5: An exponential model provides the best fit, followed closely by a quadratic model.

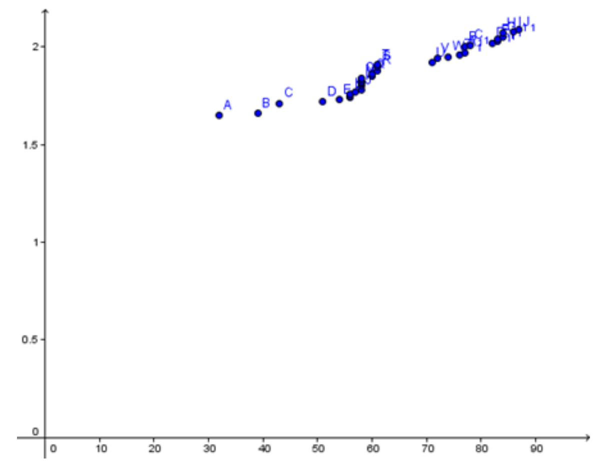


Data Set 6: The data illustrate a classic logistic model. However, students may have difficulty with the domain in trying to find the logistic model. The basic logistic model has a horizontal asymptote at y = 0. Since the data provided includes the point (0, 0) in the graph, including this point in the logistic calculation will result in a domain error.	
Data Set 7: A quadratic model provides the best fit. However, as with Data Set 1, there should be limits on the size of the particles being carried by the river.	

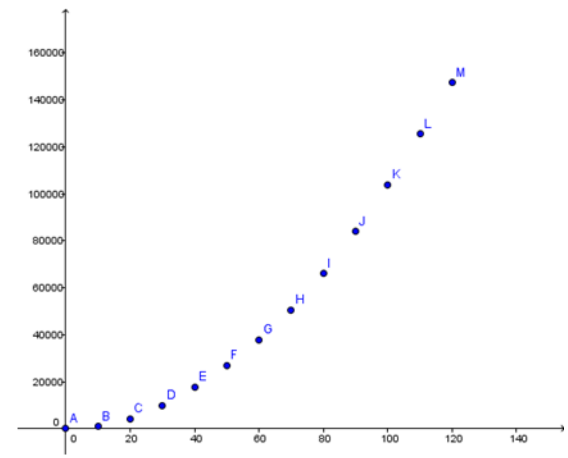
Data Set 8: A linear model provides the best fit. (Note: The given graph uses the number of years after 1900 as the x-value.)



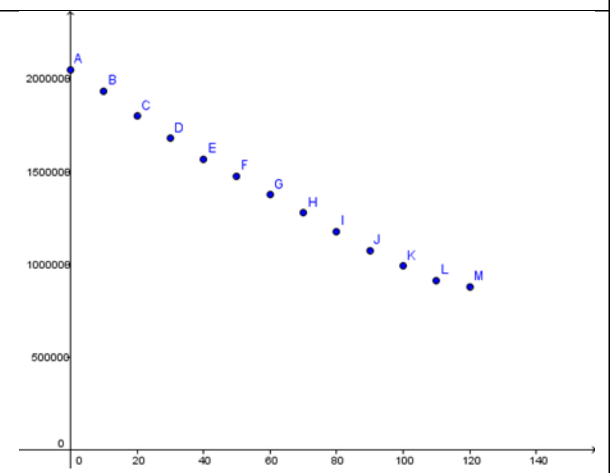
Data Set 9: A linear model provides the best fit. (Note: The given graph uses the number of years after 1900 as the x-value.)



Data Set 10: A quadratic model provides the best fit. (Note: This data is included as part of an educational project provided by NASA.)



Data Set 11: A quadratic model provides the best fit. (Note: This data is included as part of an educational project provided by NASA.)



Data Set 1: Sand Granules

For naturally occurring ocean beaches, the data below were collected. Find a function to model the relationship between the median diameter of granules of sand (in millimeters) and the gradient of the slope of the beach (in degrees).

Diameter	0.17	0.19	0.22	0.235	0.235	0.3	0.35	0.42	0.85
Gradient	0.63	0.7	0.82	0.88	1.15	1.5	4.4	7.3	11.3

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Resource: *Physical Geography*, by A. M. King, Oxford Press, England

Data Set 2: Cricket Chirps

The following data relate the number of chirps per second of the striped ground cricket and the temperature in degrees Fahrenheit. Find a function to model the relationship between these two quantities.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Chirps/sec	20.0	16.0	19.8	18.4	17.1	15.5	14.7
Temperature	88.6	71.6	93.3	84.3	80.6	75.2	69.7

(table continued)

Chirps/sec	17.1	15.4	16.2	15.0	17.2	16.0	17.0
Temperature	82	69.4	83.3	79.6	82.6	80.6	83.5

Resource: *The Song of Insects*, by Dr. G. W. Pierce, Harvard College Press

Data Set 3: Ground Water Survey

The following data were collected from a random sample of wells in Northwest Texas. The data relate the bicarbonate (in parts per million) of the well water with the pH of the well water. Use the data to develop a model for the relationship between these two quantities.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Bicarbonate	157	174	175	188	171	143	217	190	142	190	215	199	262	105
pH	7.6	7.1	8.2	7.5	7.4	7.8	7.3	8.0	7.1	7.5	8.1	7.0	7.3	7.8

Resource: Union Carbide Technical Report K/UR-1

Data Set 4: Prehistoric Pueblos

The data below relate the estimated year of initial occupation with the estimated year of the end of occupation of prehistoric pueblos in a random sample of such pueblos in Utah, Arizona, and Nevada. Use the data to develop a model for the relationship between these two quantities.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Initial Year	900	700	1125	750	1250	1250	1175	1225
End Year	1250	1300	1175	1250	1300	1280	1225	1275

(table continued)

Initial Year	1180	1080	1080	1075	1090	1225	1200	1325
End Year	1250	1150	1275	1250	1135	1275	1285	1400

Resource: *Prehistoric Pueblo World*, by A. Adler, University of Arizona Press

Data Set 5: The Size of Alligators

Many wildlife populations are monitored by taking aerial photographs. For example, the length of an alligator can be estimated quite accurately from an aerial photograph. However, the alligator's weight is much more difficult to determine. The data below show the length (in inches) and weight (in pounds) of alligators captured in central Florida. Use the data to develop a model for the relationship between the length and weight of alligators in central Florida.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Length	58	61	63	68	69	72	72	74	74	76	78	82	85	86
Weight	28	44	33	39	36	38	61	54	51	42	57	80	84	83

(table continued)

Length	86	86	88	89	90	90	94	94	114	128	147
Weight	80	90	70	84	106	102	110	130	197	366	640

Resource: Exploring Data website, http://curriculum.qed.qld.gov.au/kla/eda, Education Queensland, 1997

Data Set 6: Sunflower Height

The data below relates the height of single sunflower plant (in cm) to the number of days the plant has been growing. Use the data to develop a model for the relationship between the height and the number of days the plant has been growing.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Day	0	7	14	21	28	35	42
Height	0.00	17.93	36.36	67.76	98.10	131.00	169.50

(table cont.)

Day	49	56	63	70	77	84
Height	205.50	228.30	247.10	250.50	253.80	254.50

Resource: Reed, H.S. and Holland, R.H. (1919), Growth of sunflower seeds; Proceedings of the National Academy of Sciences, volume 5, p. 140.

Data Set 7: Moving River Particles

Rivers and streams carry small solid particles of rock and mineral downhill, either suspended in the water column or bounced, rolled, or slid along the river bed. The data below show the speed (in m/sec) necessary to carry particles in suspension as it relates to the diameter of the particle (in mm). Use the data to develop a model for the relationship between the size of the particle in suspension and the speed necessary to move the particle.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Diameter	0.2	1.3	5	11	20	45	80	180
Speed	0.10	0.25	0.50	0.75	1.00	1.50	2.50	3.50

Resource: Nielsen, A. (1950) *Oikos,* 2, 176-96 as reported in Ecology for Environmental Sciences, Anderson J.M.

Data Set 8: World Records for Men's High Jump

The data below show the world record for the men's high jump (in meters) and the year in which the world record was set (between 1912 and 1993). Use the data to create a model for the relationship between the world record for the men's high jump and the year in which the record was set.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Year	1912	1914	1924	1933	1934	1936	1937	1941	1953	1956	1957	1960	1960
Record	2.00	2.01	2.03	2.04	2.06	2.07	2.09	2.11	2.12	2.15	2.16	2.17	2.18

(table continued)

Year	1960	1961	1961	1961	1962	1962	1963	1971	1973	1976	1976	1977	1978
Record	2.22	2.23	2.24	2.25	2.26	2.27	2.28	2.29	2.30	2.31	2.32	2.33	2.34

(table continued)

Year	1980	1980	1983	1983	1984	1985	1985	1987	1989	1993
Record	2.35	2.36	2.37	2.38	2.39	2.40	2.41	2.42	2.44	2.45

Data Set 9: World Records for Women's High Jump

The data below show the world record for the women's high jump (in meters) and the year in which the world record was set (between 1932 and 1987). Use the data to create a model for the relationship between the world record for the women's high jump and the year in which the record was set.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Year	1932	1939	1943	1951	1954	1956	1956	1956	1957	1958	1958	1958	1958
Record	1.65	1.66	1.71	1.72	1.73	1.74	1.75	1.76	1.77	1.78	1.80	1.81	1.82

(table continued)

Year	1958	1958	1960	1960	1961	1961	1961	1971	1972	1974	1976	1977	1977
Record	1.83	1.84	1.85	1.86	1.88	1.90	1.91	1.92	1.94	1.95	1.96	1.97	2.00

(table continued)

Year	1978	1982	1983	1983	1984	1984	1986	1987
Record	2.01	2.02	2.03	2.04	2.05	2.07	2.08	2.09

Data Set 10: Space Shuttle Ascent Altitude

On July 4, 2006, the Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121 to begin a rendezvous with the International Space Station. The data below show the altitude (in feet) of Discovery every 10 seconds from liftoff to the separation of the solid rocket boosters. Use the data to model the altitude of Discovery at time t.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Time	0	10	20	30	40	50	60	70
Altitude	7	938	4160	9872	17635	26969	37746	50548

(table cont.)

Time	80	90	100	110	120
Altitude	66033	83966	103911	125512	147411

Resource: www.nasa.gov

Data Set 11: Space Shuttle Ascent Total Mass

On July 4, 2006, the Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121 to begin a rendezvous with the International Space Station. The data below show the total mass (in kg) of Discovery every 10 seconds from liftoff to the separation of the solid rocket boosters. Use the data to model the total mass of Discovery at time t.

a) Determine what type of model is most appropriate for the data. Explain what models were considered and why one model was chosen over the others. Explain what each of the variables represents.
b) Present graphical evidence that the model is valid for the data presented.
c) Explain any limitations of the model.
d) Create several questions that can be answered using the model. Answer the questions that are created.

Time	0	10	20	30	40	50
Total Mass	2,051,113	1,935,155	1,799,290	1,681,120	1,567,611	1,475,282

(table continued)

Time	60	70	80	90	100	110	120
Total Mass	1,376,301	1,277,921	1,177,704	1,075,683	991,872	913,254	880,377

Resource: www.nasa.gov